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Case study of a college mathematics instructor: patterns of classroom discourse

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Abstract In the United States, undergraduates—regardless of their field of study—generally must complete a mathematics course to meet breadth-of-study requirements. This report is aimed at providing a research foundation for practical efforts to improve teaching and learning in such college mathematics service courses (e.g., college algebra, liberal arts mathematics, business calculus). The case participant, Professor Kale, was a PhD mathematician with 12 years of college teaching experience, 6 years as a graduate student and 6 years after the doctorate. He and students in both of his classes agreed to the daily video recording of their meetings for an entire semester. Through constant-comparative analysis of videos and extensive interviews with Professor Kale, as well as brief interviews with his students and other members of the department, we derived a description of discursive patterns in Kale's classes. We conclude with possible implications for future work in college mathematics service course research and teaching.

Keywords Post-secondary teaching · Discourse · Pedagogical content knowledge

1 Introduction

The university experience for undergraduates in the United States includes several years of deep coursework in the area of their degree (e.g., mathematics or history) as well as "breadth" coursework in several areas outside their degree field. The mathematical content

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of courses offered in service to breadth requirements varies from beginning algebra and elementary statistics-based courses, to analytic geometry and trigonometry content, to a first course in calculus. Instructional staffing for service courses also varies by institution: from almost all being taught by graduate students with bachelor's degrees in mathematics, to most being taught by people with advanced degrees in mathematics. Many in the US who teach them are unaware that service course enrollees may not share mathematicians' views about mathematics and may never have experienced mathematics as interesting or clear (Hauk, 2005; Ouellet, 2005). Of the 15 million undergraduates in the US each year, 85% take mathematics service courses such as the college algebra course taught by the PhD mathematician in this study (Horn, Peter, Rooney, & Malizio, 2002). The national average pass-rate for these courses hovers around 60%, the other 40% either withdraw or fail; also noteworthy is that half of US students who declare mathematics and physical science majors switch to other fields—with 90% citing poor teaching as a reason (Seymour, Melton, Wiese, & Pedersen-Gallegos, 2005).

Improving college mathematics teaching can productively start with ways to build instructional self-awareness through opportunities to compare and contrast to other people in other contexts (Mason, 2009). Towards that end, the research reported here is a case study of the classroom interactions of a PhD mathematician teaching in a mathematics service course, college algebra, at a large research university in the United States.

1.1 Note on mathematics service courses

For those readers unfamiliar with practices at large US universities, *College Algebra* is typically a one-semester course based on developing the concepts of variable (through work with polynomial equations) and function (through work with multiple representations of polynomial, rational, exponential, and logarithmic functions). At many US institutions, College Algebra satisfies the mathematics breadth requirement while some universities require a semester of calculus. At large universities, where thousands of students will take the same course with many sections taught by many different instructors, *course coordination* is a process whereby one (or more) people oversee the instruction for all class sections. Here, the term "class" refers to a group of people in a room and "course" to an administrative structure that may consist of many classes taught by many different instructors, all with the same base content. Course coordination can be as loose as instructors getting together and agreeing, verbally, to particular big ideas as targets for instruction or as structured as a common timeline and syllabus with common assessments given on common days in all sections of a course.

1.2 Note on teaching assistants (TAs)

In US universities, a graduate student who has a teaching assignment is most often called a "teaching assistant" or "TA." The three most common forms of TA assignment are:

1. Support for a large lecture—for example, 200 undergraduates attend a large lecture given by a professor for 3 hours per week which is supplemented by five separate 1 hour per week problem-solving sessions, each attended by 40 students and each led by a different TA.
2. One-on-one tutor to undergraduates (usually in a tutoring center or "lab"), or
3 An "instructor-of-record" who has the same instructional responsibilities as a faculty member, such as writing a syllabus, instructing during class meetings, holding office hours, and writing and grading assessments.

2 Theoretical framework

The perspective behind the design of the study is a constructivist interpretation of social cognitive theory (Bandura, 1986). From this view, human interaction can be described in terms of the *personal*, *behavioral*, and *environmental* factors involved in cognitive, affective, and social activity. Within this perspective, we have built a framework for data analysis and reporting about college instruction that relies on three grain sizes: (1) the professional context of educational curricular values, (2) the local discourse of communication in college mathematics classes, and (3) on the level of the individual, the pedagogical content knowledge of the instructor. The research questions address these three aspects, and this report offers a focus on the second grain size: communication patterns in a college mathematics classroom. As a result of this focus, the content of the course is not in the foreground in this report; rather, it is the patterns of communication for making sense of mathematics and sociomathematical aspects of student engagement with mathematics in the college classroom.

2.1 Professional context

Common views of instruction in US colleges and universities fall into four broad categories (Davis, Hauk, & Latiolais, 2009; Grundy, 1987): transmission, product, process, and praxis. We do not argue a value-laden hierarchy to the models—each can be useful given the cultural (personal and behavioral) factors of teacher and students in an instructional environment. The transmission model approaches curriculum as the content of the syllabus and textbook and values a view of instruction as the act of speaking (transmitting) the content. In the product model, curriculum is a set of goals about knowledge acquisition where uniform assessable objectives and associated test performance by students are valued. In the process model, curriculum is the set of materials that supports the process of developing thinking skills and the primary value is found in each student in the room learning something. Within the praxis instructional paradigm, curriculum is valued as the collective practice of teacher and students engaging with the world through knowledge of mathematics and other content.

Mathematics is shaped just as much by human and cultural knowledge as any other field, from the values of society that are implicit in classroom interactions (Bishop, FitzSimons, Seah, & Clarkson, 1999) to the culture of an institutional community. In collegiate mathematics instruction, as in school teaching contexts, there are myriad overlapping explicit and implicit value sets. In each of the four models of curriculum and instruction, one can identify ways of valuing within the expected, intended, implemented, and achieved curricula (implicit or explicit). For example, in collegiate mathematics service courses in the US where the dominant paradigms are a blend of the transmission and product models, the expected curriculum is represented by the content of the university-approved syllabus and the middle-class socio-cultural expectations for classroom behavior supported by the university environment. Intended curriculum in these courses has at least three components: (a) the content to be learned as asserted by the department (which may be mediated by course coordination), (b) the mathematics the instructor intends students to learn, and (c) the content intended by the authors of the text. The implemented curriculum is what actually occurs in a classroom, the mathematical opportunities to learn enacted by the teacher and students. In product-model-based mathematics service courses, the achieved curriculum is the mathematical ideas, signs, and their relationships that students demonstrate knowledge of on examinations during and at the end of the course.

2.2 Mathematical classroom discourse

We use the term discourse to mean "connected stretches of language that make sense" (Gee, 1996, p. 127) to those involved in producing it (e.g., speaking) and taking it in (e.g., hearing). The patterns of discourse we explore come from spoken (and at times gestural) language that happened in a college mathematics classroom. We distinguish this from the kind of cultural repertoire called Discourse (with a capital D) that involves rules, values, artifacts, and a variety of linguistic and behavioral markers for "identifying oneself as a member of a socially meaningful group … or to signal (that one is playing) a socially meaningful role" (Gee, 1996, p. 131). Future work will explore how classroom Discourses might come from and shape the types of discourse patterns discussed here (for more on discourse-related research in mathematics education, see Ryve, 2011).

The teacher *initiation*—student *response*—teacher *follow-up* or *IRF* triadic structure is a common pattern of classroom discourse (Cazden, 2001). Multiple disconnected *IRF* interactions are a discursive pattern that may dominate even in inquiry-based instruction, yielding a teacher-regulated kind of interaction that does not include deep participation by students. For example, follow-up moves to students' responses might be used for knowledge transmission instead of inviting students to contribute ideas for knowledge construction (Nassaji & Wells, 2000) or opening opportunities for students to be agents in mediating the actions of learning (Wertsch, 1998). Truxaw and Defranco (2008) refer to a single *IRF* exchange as univocal discourse and to cycles as dialogic. That is, *IRF* interaction that is recursive in this sense is dialogic: $\{I_k \quad R_k \quad F_k\}_{k\geq 2}$ where I_{k+1} depends on R_k or F_k or their respective contexts. The authors also suggest that dialogic discourse has the give-and-take communication needed to promote student self-regulation in learning. Moreover, note Truxaw and DeFranco, verbal moves in a recursive *IRF* model involve revisiting the frame of reference "in ways that situate it in a larger context of mathematical concepts" and that foster students' "mathematical meaning-making" (p. 514).

Just as Yackel and Cobb's (1996) work in second-grade classrooms on sociomathematical norms, "such as implicit understandings of what constitutes an acceptable mathematical explanation and the means by which technology can support a mathematical explanation" (Yackel, Rasmussen, & King, 2000, p. 276) has resonated in research on teaching and learning in collegiate differential equations, we see clear connections between Hufferd-Ackles, Fuson, and Sherin's (2004) work on "math-talk" in a third-grade classroom and our experiences researching and teaching undergraduate mathematics service courses. To develop a successful "math-talk learning community" in which self-regulation by students and scaffolding by the instructor create an environment conducive to learning, Hufferd-Ackles and colleagues (2004) reported on the importance of students acting in central or leading roles in discourse. They identified four levels of discursive interaction, defining each level by kinds of student and teacher engagement in (a) questioning, (b) explaining mathematical thinking, (c) being a source of mathematical ideas, and (d) making sense of mathematical arguments. It is worth noting here that the kinds of sociomathematical norms established in (a) through (d) play an essential role in both talking about mathematics and talking about the workings of a mathematics class. In developing from Level 0 to Level 3 in the math-talk model, the locus of control for questioning, explaining, reasoning, and sense-making shifts from the teacher (Level 0) to shared by students and teacher (Level 3). The foundation of the levels of math-talk is Level 0, exemplified by instructor-only speech and students who are responders in rote ways to teacher elicitation (i.e., mostly *I* and *R,* but little *F* in discourse). Level 1 involves one-step and some brief two-step iterations of *IRF* interactions (i.e., not recursive in the sense of Truxaw and Defranco (2008)); at Level 1, a teacher-generated I_2 may depend on R_1 or F_1 with little reference to mathematical framing or connecting of ideas. Level 2 has recursive $\{I_k \quad R_k \quad F_k\}_{k\geq 2}$ interactions between teacher and student(s) that include students responding to each other, perhaps through the teacher, and where students "begin to stake a position," "listen supportively" to each other, and "student ideas sometimes guide the direction of the math lesson" (Hufferd-Ackles et al., 2004, p. 89). Level 3 includes recursive interaction between teacher and students *and among students*—both within student groups as well as during whole-class interactions.

2.3 Pedagogical content knowledge

Mathematics pedagogical content knowledge (PCK) is needed or used by an instructor while planning, implementing, and reflecting on teaching. PCK for college mathematics instruction is related to subject matter knowledge in that it draws on the foundations of mathematical approaches to thinking (e.g., reasoning, proof, and problem-solving) but is different from such content knowledge in that it involves using these ideas in the context of working with people (rather than in working with mathematics). PCK includes knowledge about formal and informal mathematical discourse and at its core are (a) a teacher's anticipations regarding students' engagement with curricular content (including confusion) and (b) how to turn teacher intentions into actions (Ball & Bass, 2000; Shulman, 1986, 1987). These aspects of PCK contribute to enacting the implemented curriculum in effective alignment with the expected curriculum.

2.4 Research questions

In this report, our primary interest is to document the classroom discourse between students and instructor over the course of a semester of college algebra. The main questions addressed here are as follows:

1. What is the nature of classroom discourse, and patterns in discourse, for this instructor in these two college algebra classes?

   Additionally, in reporting on the above, we address two other areas of interest—these are also foci to be explored in greater detail in our future publications:
2. How does the professional environment, particularly course coordination, interact with classroom discourse?
3. How does the instructor's pedagogical content knowledge reflect and get shaped by the classroom discourse in the classes he teaches?

3 Methods

3.1 Participants and setting

The main participants were the instructor, Professor Kale, and the 70 students in his two college algebra classes. These classes were two of more than 50 sections taught that semester at Big Research University (BRU), a state-funded school with a full-time student enrollment of over 50,000 and more than 100 mathematics faculty. There were really two mathematics departments at BRU: one for tenured and tenure-track faculty (all with doctoral degrees) and a separate departmental office for service course mathematics (i.e., courses with mathematical content at and below the level of first year calculus). The instructional staff for service courses consisted of about 30 long-term-faculty (renewable contract employees without the job security of tenure; half with doctorates, half with master's degrees) along with about 30 TAs who were instructors of record for the classes they taught. Each multi-section service course was "coordinated" by one or two of the long-term faculty members who were also teaching in it (see Section 4.2).

3.2 Data collection, analysis, and coding

After obtaining informed consent from students on the first day of classes, the third author or a research assistant video recorded each of Professor Kale's class meetings. The third author also generated field notes from interviews with Professor Kale throughout the term. A year later, after initial coding of the data, all of the authors conducted a video-clip interview with him. Additional member-checking interviews and email communications with Professor Kale took place during the development of the manuscript. Reported here are the results of constant-comparative qualitative analysis of classroom video recordings and contextual detail from interviews with Professor Kale. Four additional sets of data informed the study and provided background information included in this report:

1. Video recordings and field notes from two class meetings that were taught by a graduate teaching assistant, Mike, when Kale went to a conference,
2. Field notes from interviews with course coordinators,
3. Field notes from interviews with Kale's students, and
4. Observational notes on six other instructors teaching college algebra at BRU.

We (re)viewed and coded ten class meetings selected from the 88 meetings captured on video in Professor Kale's two classes. We analyzed the first two meetings for both classes in week 1 and one meeting for each class in weeks 5, 9, and 14. Individuals and pairs of researchers coded each of the selected classes. Every coded result was checked by at least two researchers, and final codes were reached by discussion and consensus. We developed a time-series method for depicting the audit trail of classroom interactions among the categories identified in coding. Categories and associated dimensions are provided in this section. How categories combined into four discourse patterns is illustrated in Section 4 with time-series diagrams and classroom transcription analysis. The first two of the seven categories had distinct continua (Figs. 1 and 2); the other five—related to attributions for responsibility and effort—all shared the same locus-of-control continuum (Fig. 3).

3.2.1 TPK

What Professor Kale referred to as his "knowledge of the content area background of students"—knowledge of mathematics and its teaching and learning that students brought with them to the course—we named *teacher's perception of knowledge* (abbreviated TPK in figures). This construct was categorized as a component at the personal node for Professor Kale, a part of the cognitive capital he brought to the class. In particular, it was part of his pedagogical content knowledge, as anticipatory knowledge. The continuum for identifying the nature of the teacher's perception of knowledge went from what the teacher expected

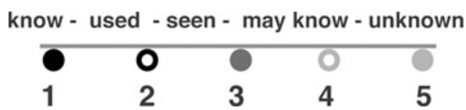


Fig. 1 Continuum for coding teacher's perception of student knowledge (TPK)

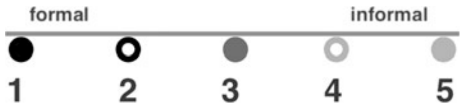


Fig. 2 Continuum for coding student and teacher perception of relationships (SPR and TPR)

students "should know" to those things he "didn't really expect them to already know when they walked in the room" (the "unknown"; see Fig. 1).

3.2.2 SPR/TPR

We noted a category for student and teacher *perceptions about the nature of social relationships* within the classroom (abbreviated SPR and TPR in figures). This environmental node category was about certain types of culturally mediated interactions. The continuum went from formal, highly structured, and "teacher-as-authority"-based relationships (e.g., students raised hands before speaking) to informal, unstructured interactions as might be exemplified by a student speaking directly to another student or to the class as a whole (see Fig. 2).

3.2.3 TAR/SAR

In reviewing classroom video, we identified four subcategories of classroom interaction that involved *attributions of responsibility*, that is, determining whose job it was to do or know something. The attribution of responsibility subcategories were:

1. *Ownership*: Whose job it was to know and understand (TAR-O and SAR-O). We identify this category as a college-level version of Hufferd-Ackles and colleagues' (2004) category (c), "being a source of mathematical ideas" (p. 87).
2. *Sense-making*: Unpacking or mathematizing (Lesh, 1996) of ideas to make sense of a concept, by teacher and by students (TAR-SM and SAR-SM). We see this as a collegiate classroom parallel to Hufferd-Ackles et al.'s (2004) (d), making sense of mathematical arguments. However, unlike third-graders, "sense-making" done out loud by adult college students is enacted by learners who have ten additional years of experience in communicating with mathematical terminology when asking questions about, and explaining their approaches to, mathematics problem situations. As a result, classroom norms regarding what was mathematically appropriate in communicating publicly in the classroom, such as in Hufferd-Ackles et al.'s (2004) (a) questioning and (b) explaining thinking, were coded into the categories TAR-SM and SAR-SM.
3. *Behavior*: Determining and engaging in classroom-appropriate social behaviors by teacher and by students (TAR-B and SAR-B). This category captured social behaviors including norms regarding communication in the classroom, such as the establishing of a behavioral norm of hand-raising and turn-taking in sharing ideas during whole-class interactions.

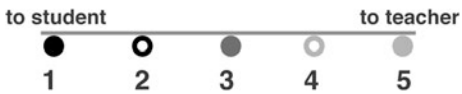


Fig. 3 Continuum for attributions (locus of control "held by student"to "held by teacher")

4. *Text*: Textual clarity and interpretation for the textbook, student written products, and teacher written products; these were attributions made by the teacher (TAR-Text);

The continuum for each of these was for locus of control of responsibility along a continuum (see Fig. 3) from "attributed to students" (dark disks in figures) to "attributed to teacher" (light disks in figures).

3.2.4 TAE/SAE

A separate category was that of *attributions about effort* by teacher and student (TAE and SAE). Like the category TPK, Professor Kale's attributions about effort were associated with PCK and his anticipatory knowledge. Also part of this category was a sub-category specific to teacher attributions about the effort needed for using technology. Like the attribution of responsibility subcategories, the coding continuum was from a locus of control of effort attributed to the student at one end and to the teacher at the other (see Fig. 3).

Note on terminology: In reporting the results, we use "most," "usually," and "typically" to refer to something occurring in at least 50% of observations and "some," "sometimes," and "occasionally" to events or characteristics appearing in fewer than 50% of observations.

4 Results

First, we introduce Professor Kale. Then, we give background on course coordination at the university. The remainder of the section is devoted to describing and exemplifying the four main patterns of discourse that emerged from coding. We also provide a brief comparison of discourse patterns between Professor Kale and TA Mike, a graduate student novice instructor who taught Kale's classes one day. As with Professor Kale, all names are pseudonyms; quotes come from Kale unless otherwise indicated.

4.1 Professor Kale

At the time of the study, Professor Kale was a PhD mathematician with 12 years of college teaching experience (6 years as a graduate student and 6 years since the PhD). He completed his PhD in mathematics at a major research university in the US. His teaching experience in graduate school was mostly as a teaching assistant who led weekly 1 hour problem-solving sessions with groups of 30 or so students; these sections were in addition to the 3 hours of lecture given by a professor each week. However, after he had passed his PhD qualifying examinations, Kale was given summer courses to teach as the instructor-of-record. As was the common experience at the institution, he had never been a student in the service courses he was assigned to teach. For these summer courses, the department provided a syllabus, a textbook, and a TA to lead an associated problem-solving session.

Kale remarked that the classes he taught as a graduate student were racially and ethnically diverse, "I had to learn early on that I needed to be able to say the same thing, the same mathematical idea, in several ways, to communicate to everyone, Asian, Asian American, African American, European, European American, Latino, and Latina, in the room." When Kale was finishing his PhD and doing his first job search, the unemployment–underemployment rate for new mathematics PhDs in the US was nearly 30% (Davis, 1997). Like many other new PhDs, Kale had little guidance from faculty at his university in applying for jobs. He sent out dozens of generic applications for mathematics research-focused jobs at major doctoral-granting universities and for teaching-focused jobs at non-PhD-granting institutions. For 4 years, he worked in temporary 1-year and 2-year positions, teaching from two to five classes per semester, as he continued to publish mathematics research. In the weeks before the study, Professor Kale received and accepted an offer from BRU. He was not familiar with the region of the country or the university. His new job was as a visiting faculty member assigned to teach four service classes, each with approximately 35 students.

From Professor Kale's interviews and teaching episodes, we saw that his intentions included fostering students' autonomy by having them take responsibility for sense-making. Another of Professor Kale's intentions was to involve students in discourse, in a belief that students supported to engage in discussion would learn from it.

> *Kale (interview):* I really try to make them [students] feel involved in the class. I really try to make students feel, you know, we're all here together. .... In my experience, you know, students know things. Sometimes students are confused by certain things, sometimes students might know something on a non-mathematical level.

He respected students' as intellectual beings and sought to learn about their sense of ownership of mathematical knowledge from what they said and asked. One of the ways he communicated this implicitly at the start of the semester was by asking for and then restating student names. One of the ways he communicated this explicitly throughout the term was by asking pointed questions, such as: "Do you understand what I mean by {mathematical concept/term}?" Kale's implemented curriculum included invitations to participate in discussion that obliged students to explain their thinking and make sense of other's explanations, including the teacher's.

4.2 Course coordination

At BRU, college algebra was a service course satisfying the university's breadth requirement for those undergraduates not going into a science, technology, engineering, or mathematics field. The course coordinators were two men with PhDs in mathematics, Pat and Lee. According to their interviews, coordinators had four responsibilities: (1) write syllabus and exams; (2) call meetings of instructors to get feedback on drafts of exams and course policies; (3) report to the department director on exam and course pass rates; (4) supervise the service hours of the college algebra faculty (e.g., instructors' work in the tutoring and testing centers). Interviews with the course coordinators indicated that Pat was a long-time adherent of a lecture-based, common-exam-dependent version of the product model and that Lee preferred a transmission-focused approach to lecture-based instruction in service courses. Though Lee said he had at first found his students' performance on common exams disappointing, he reported that he had become comfortable with common exams and that his students' performance had improved since he had become course coordinator and an author for the common exams.

One of the departmental policies supported wholeheartedly by both Pat and Lee was the use of a testing center. All service course students took three common exams outside of class on prescribed days in the testing center; this was a large lecture hall (200 seats) where a doorkeeper swiped a student's identification card for entry and exit. Instructors were required to be proctors; by coordinator policy, instructors stood at the door of the testing center to check calculators for, and erase, formulas as students entered. The coordinators' "anti-cheating" calculator policy led to unpleasant interactions for several instructors, including Kale. He felt an obligation to obey the course coordinators' design for course assessment by enforcing their "no formulas" policy. However, Professor Kale was uncomfortable in the authoritarian role the policy placed him with students. He felt the use of the center was "stressful for the students" because they were tested away from the familiar classroom environment. In his previous teaching positions, Professor Kale had the autonomy of writing and giving his own tests and establishing his own policies.

4.3 Classroom interaction patterns

Four regularly recurring patterns emerged from diagramming the coded class meetings in time-series. These four patterns accounted for more than 90% of the interactions we observed. Note that these patterns were determined from coding 10 of 88 hours of video. A later random selection and viewing of two recordings (one from week 2 and one from week 7) did not falsify the regularity of occurrence of the patterns. The four patterns are discussed and exemplified below:

Pattern C Professor Kale's lecture pattern; approximately 65% of class time;

Pattern A Sense-making and negotiation of sociomathematical norms (whole-class or while students worked in groups, pairs, or individually); approximately 25% of class time;

Pattern B Negotiation of social norms (e.g., for authority) mediated by the course coordinators' expected curricular values; approximately 5% of class time;

Pattern D Conflict escalation and resolution; approximately 5% of class time.

As an illustration, and for later reference, Figs. 4, 5, and 6 show the coded interactions for a total of 22 min of Professor Kale's classes and Fig. 7 shows interactions for 8 min of TA Mike's time with the class. Figures 4 (Pattern B) and 5 (Patterns A and C) together record a 15-min chunk from the second class meeting in the first week of the term. Figure 6 is an example of Pattern D from the penultimate week of classes. The positioned circles and disks in the figures encode three things:

- *Category*. The vertical position of disks indicates the coded category for the utterance (see the key at the left of each figure) with teacher utterances above the horizontal "axis" and student utterance codes below; the horizontal position of a disk indicates the temporal position of the utterance. For example, the coding in Fig. 5 goes from minute 13 to about minute 20 of a class meeting.
- *Locus of control*. The shade and open/closed properties of disks indicate our interpretation of locus of control attribution by the speaker, darkest is locus of control attributed to student(s), lightest is locus of control attributed to the instructor (a D indicates the attribution is to the Department coordinators). For example, the light disk with a D at its center at about 5:20 in Fig. 6 represents an assertion made by a student about departmental policies at about 5 min into the class meeting.
- *Connection*. Lines join pieces of discourse that seemed to have been connected for the speakers. For example, in Fig. 4 (section B2), the two lines from the open disk about 5 min, 20 s into class represent two different student responses to one of Professor Kale's statements.

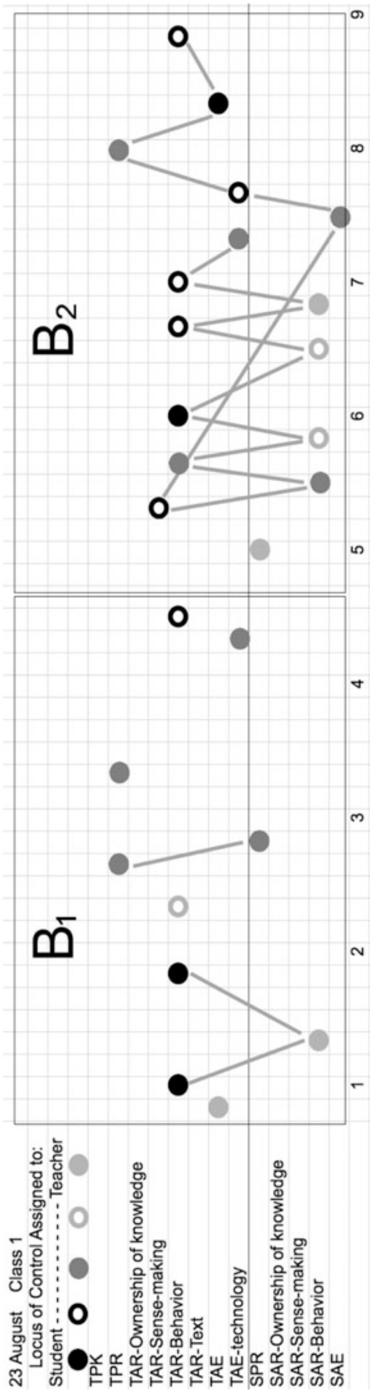


Fig. 4 Pattern B, two iterations, time-series coding representation of first 9 min of class, week 1, meeting 2

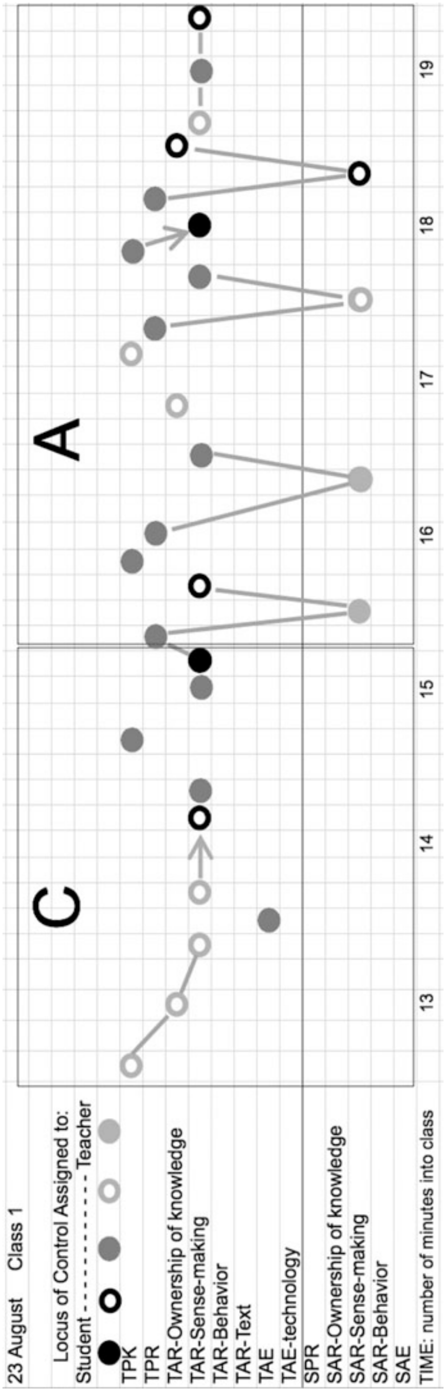


Fig. 5 Patterns A and C, continued time-series representation of second 10 min of class, week 1, meeting 2

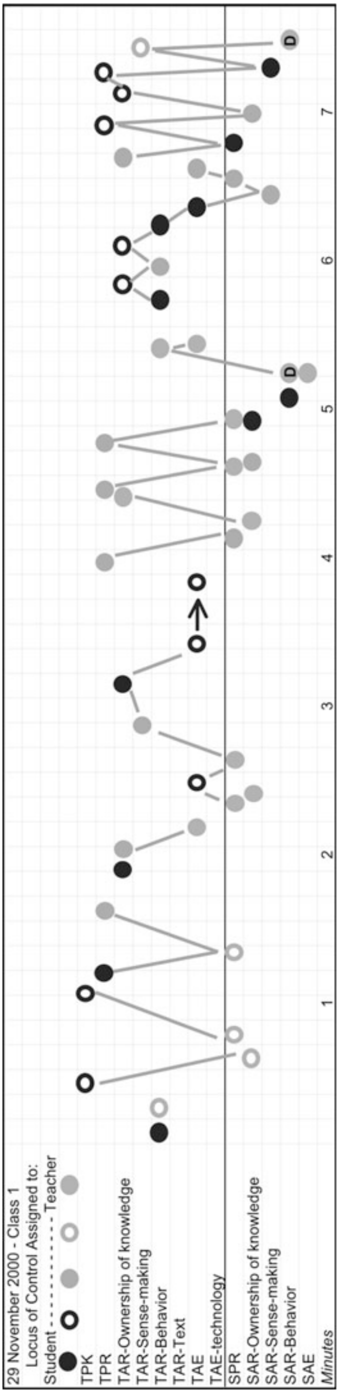


Fig. 6 Pattern D time-series representation for first 8 min of class, week 14, meeting 2

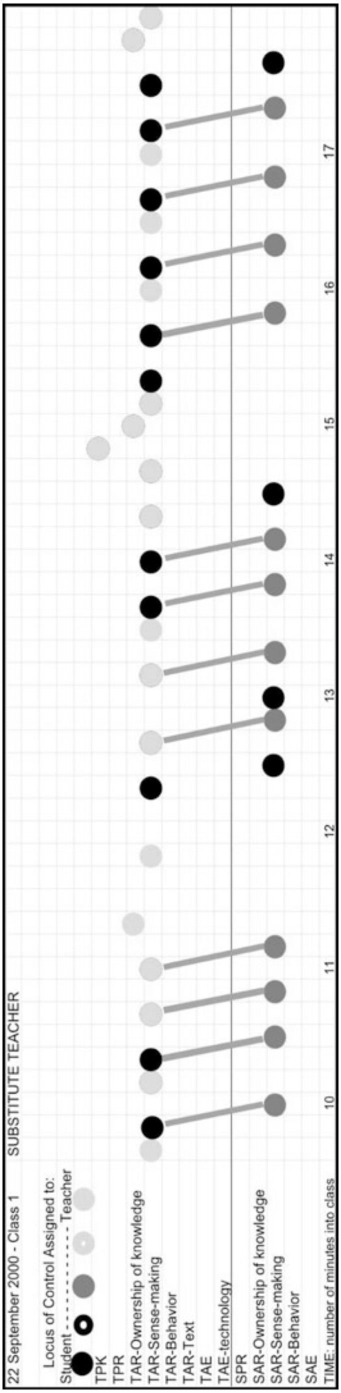


Fig. 7 Time-series representation for TA Mike, teaching Professor Kale's class (same students as in Figures 4, 5, and 6)

To give the reader a sense of context, the piece of class meeting illustrated in Fig. 4 consisted of what Professor Kale called “announcements and housekeeping” (minutes 1 to 11), and the coding shown in Fig. 5 concerned a bit of lecture and guided discussion. For example, in minutes 13 to 14, the coded interaction is of Kale’s attribution that “the teacher owns and is responsible for mathematical knowledge and sense-making” then, after some back and forth with individual students, this changes to “students must somehow make sense of this” (minutes 18 to 19). Note that, in Fig. 5, the student responses (below the horizontal line) evolve from full attribution of responsibility to the instructor to some responsibility attributed to the student. That is, Professor Kale checked regularly, through questioning and eye contact whether the sense-making for classroom content and processes achieved by students coincided with his intentions.

In describing the four patterns of discourse, we start with the Pattern C, the pattern that was most frequent.

4.4 Pattern C: Lecture

Professor Kale’s classroom style was to present lecture chunks, each chunk concerning a particular concept or problem-solving method. These pieces of lecture ran for as little as 2 min up to 20 min and generally followed the pattern shown in Fig. 5 (12:30 to 15:00). Pattern C usually was followed, in turn, by the interactive discourse of Pattern A.

Professor Kale’s expository presentation of material, Pattern C, typically began with a statement of where in the text the work in question was located and a reference to his perception of student knowledge. That is, he made explicit assertions about his own anticipatory knowledge. Professor Kale noted in his interview that he had learned early in his service course teaching that “everyday mathematical vocabulary” was often “foreign” to students. During this class, he voiced his perception that his students may never have heard of “mathematical modeling” though he saw it as common knowledge for a mathematician (Fig. 5, minute 12):

> *Kale*: We’re looking at Section 1.2 of the textbook, which is called “Mathematical Models”. Now you may not have heard those words used that way, “mathematical models” when, in fact, this is really what math is, the reason that math is studied (TPK 4).

In Pattern C, what usually followed his opening statement of purpose and an observation about his perception of students’ familiarity with a topic was an assertion about the nature or history of the concept and an exhortation to make sense of it. A significant intended curricular value for Professor Kale was that students engage in sense-making. In Fig. 5, he first indicated a great deal of ownership of the knowledge for himself and the mathematics community (TAR-O 4) and that the students would be expected to make sense of it with his help. In Pattern C, Professor Kale relied on what he called in an interview, “virtual dialog,” speaking as if anticipating responses from students and then answering those potential or “virtual” responses. He continued,

> *Kale*: That is, what you try to do is take a real world problem and find a way of changing that problem into mathematical statements, usually algebraic statements, that involve variables because it makes it easier to resolve those kinds of statements. The history of mathematics, at least for the West, is one of refinement of notation because there are many problems that were understood but were very hard to solve because the notation was so horrible. (TAR- SM 4)

Professor Kale often introduced main mathematical concepts in a lecture chunk through one or more examples. Early in the semester, he did some scaffolding for the sense-making and asked students to help him complete the example task. Later in the term, he more rapidly completed an example, with occasional checking with the students to "make sure they were following." The initial guided sense-making was usually followed by Professor Kale asking students to work together on a similar problem as he helped individuals and groups (see Section 4.5). Continuing the piece of lecture already begun above, he said:

> *Kale*: So, let's actually work on some problems directly from the book. So I'll do a few here as examples on the board for you, and then I'll let you work on a few in class together today before I give you your homework assignment. So, let's start with, uh, number one on page 19. Okay, so on page 19, number one, we're asked to "write an algebraic expression for the verbal expression." If you have your book, you don't actually have to write it down. I'll write it on the board in case you don't have your book. In general, you'll bring your book to class every day. [This aside about effort to bring books to class is indicated by the TAE 3 at about 13:50 in Fig. 5]. And here is the verbal expression: "The sum of two consecutive natural numbers" (writes on board as he says it). The first thing you should notice, especially if you're an English major, is this is not a sentence: "the sum of two consecutive numbers, two consecutive natural numbers." There's no verb there, it's simply a phrase. It's an expression. "The sum of two consecutive natural numbers" we're not saying that it is anything. We're just trying to find a way of translating it, that statement, from English, into mathematical notation. So, can anyone help me here? (Student raises hand) Yes? Please tell me your name as well.

At the point where students joined in to a sense-making or problem-solving activity, the pattern of discourse changed from the one identified as Pattern C to Pattern A.

4.5 Pattern A: Sense-making and negotiation of sociomathematical norms

This pattern captures Professor Kales' negotiation with his students for encouraging what he desired as classroom sociomathematical norms. Pattern A is exemplified in the coded transcription shown in Fig. 5 from 15:20 to 20:00. During this piece of class time, Professor Kale engaged his students with what was meant by "mathematical models." More generally, Pattern A interactions were characterized by four components:

1. Kale encouraged students' participation and discussion with him (Initiation).
2. Students responded to him (Response).
3. Kale verbally rephrased or reorganized students' representations/connections (Follow-up).
4. Kale encouraged students to reason or debate about the representations and/or connections with each other ([re]Initiation connected to follow-up).

Sometimes, Professor Kale asked students to move their desks to face each other in small clusters and gave them problem tasks to work on in groups (in chunks of 10 to 20 min of class time). In these cases, Pattern A would occur locally in the group space as he moved around the room checking in and talking over the problem with each group. This local, in small groups, use of Pattern A accounted for approximately 40% of Pattern A instances.

From Professor Kale's video-clip interview about the whole-class Pattern A interaction given below, we identified curricular intentions for soliciting and validating of students' understanding. Kale asserted in his interview that he wanted to "share the responsibility" of "sense-making" and make the in-class learning experience for his students different from what it may have been for them in a "straight lecture" setting. Professor Kale's interactions in Pattern A were indicative of how he used his experience that "students know things" to negotiate responsibility for sense-making efforts during instruction.

Continuing the example, Professor Kale encouraged students to participate in discussion by asking, "So, can anyone help me here?" He initiated the responsibility of sense-making as a shared effort. Some students volunteered to participate:

> *Kale*: (at 15:50 in Fig. 5; Kale gestures to a student whose hand is raised) Yes? Please tell me your name as well.
> *Student*: Oh, Dan.
> *Kale*: Dan?
> *Dan*: Yeah. Uh, put like y equals x plus x plus one.
> *Kale*: Okay, so Dan says, why don't we put y equals x plus in parentheses x plus one [Kale looks to Dan for confirmation and appears to receive it]. Okay, so what do you think (TAR-SM 2)? This is Dan's suggestion (TPK 3). (pause) Okay, I see some people nodding yes, and some people nodding no. Okay, well, those of you, someone who's saying yes, why do you think this is correct? [A student raises her hand (SAR- SM 1)]
> *Kale*: Yes, please tell me your name.
> *Student*: I'm Annie, Annie.
> *Kale*: Annie.
> *Annie*: Because x equals a number and then you're going up one so you can say that number plus one.

In some cases, Professor Kale anticipated that "some of the students might not have mathematical knowledge they needed" in a problem situation. In such cases, he took more responsibility for knowledge ownership. He noted during his interview that as the interaction (begun above) continued, he remembered feeling he should provide some information about the natural numbers and the meaning of the word "consecutive."

> *Kale*: (16:40, responding to Annie) Okay, so we have a number, x, and then we're going up one. Here's where the "consecutive" comes in. Consecutive means next. So, we have a number, and then we have the next number. We can assume that x is standing for a natural number. Do you understand what I mean by natural number? Do you know what natural numbers are [looks around the room] (TAR-O 4)? The natural numbers are the positive integers. They're numbers 1, 2, 3, 4, etcetera. Not fractions, not negatives, not decimals, but 1, 2, 3, 4, 5. The counting numbers. The numbers you usually refer to. The numbers you use when you're very small (TPK 4). And so we have x plus x plus one. [Another student raises her hand] I saw a hand here (TPR 3), what was your name?
> *Student*: Um. Winona.
> *Kale*: Winona.
> *Winona*: I was just wondering how you were going to specify that they were all natural numbers (SAR-SM 2)?

Winona's inquiry indicated that she took some responsibility for sense-making. After responding to Winona's comment, Kale asked for input from those who disagreed with Dan's suggestion. As Yackel and colleagues (2000) noted, "norms are based on expectations and obligations that are constituted as participants interact with each other" (p. 281). In the case here, whether Dan's answer counted as an acceptable mathematical explanation depended in part on his classmates' responses:

> *Kale*: Okay, now. There were also some people who were shaking their head no and didn't think this was right. So, can I have a comment from one of those people? Yes, and your name is?
> *Student*: Ruth, and uh,
> *Kale*: Ruth.
> *Ruth*: Um, do we need the *y*, or are we just looking for an expression?
> *Kale*: So, what Ruth says is "do we need the *y*?" Well, notice what this statement says, we have "a number is equal to the sum of two consecutive natural numbers." So notice that now we have a sentence that does have a verb in it. That is, we're actually saying that these two things are equal.

Professor Kale had previous experience with teaching algebra and had anticipated that students might render an expression as an equation. Hence, he noted in an interview, his "purposefully remarking on it early in order to be able to refer back to the idea of a verb."

The distribution of levels of interaction in Pattern A. While Pattern C was similar to the Level 0 discursive interaction described by Hufferd-Ackles et al. (2004), Professor Kale's reliance on "virtual dialog" during Pattern C is like Level 1 in the sense that he voiced *I*, anticipated *R*, and then voiced *F*. Also, given his conversational invitations, he appeared to communicate his expectation for establishing a more complex interaction as socio-mathematical norm. In the example Pattern A, Professor Kale's gathering and revoicing of student views promoted two iterative *IRF* discourse cycles (a Level 1 interaction). From this opening of discourse, a second cycle of *IRF* was initiated by a student, to evaluate another student's mathematical ideas, though it still passed through Professor Kale's revoicing (a Level 2 cyclic *IRF* interaction). On the longer time scale of the whole semester, we also saw Level 2 interaction when the class worked in small groups. In fact, over the semester, Pattern A interactions went from mostly Level 1 and 2 to include some Level 3 in student group work settings (though not in whole-class discussions) by mid-term, then after 2 weeks fraught with Pattern D interactions (see §4.7), settled back to mostly Level 1 and Level 2 after week 10 (see Fig. 7).

Note on the two class sections While Fig. 7 summarizes both class sections, it is worth noting that the two sections had slightly different profiles. The two classes met consecutively. Professor Kale usually arrived 5 min before the first section began, left with students at the end (another class came in) as he headed for the second room. He arrived in the second room 5 to 7 min before class. From the very first day of the term, there appeared to be more student-to-student interaction in the first section during the time just before class began. Students often talked to each other in pairs and small groups. Close review of the recordings revealed that some of the conversations were about mathematics, but it is impossible to know if this was true for all of the conversations. This type of student-to-student direct interaction did not begin in the other section until about the sixth week of classes. By this time in the semester, both classes exhibited private student-to-student interactions during class and most of these conversations were directly related to the mathematical content of the lecture or in-class work.

4.6 Pattern B: Negotiation of social–behavioral norms

Pattern B recurred regularly throughout the semester during Professor Kale's "housekeeping portions" of classroom interaction. Pattern B involved negotiation of classroom social norms based on his expectations regarding student behavior and his intended curricular values. Minutes 5:00 through 11:00 in Fig. 4 provide an illustration of Pattern B as it most commonly occurred in the first half of the term.

In particular, the second of the two Pattern B negotiations in Fig. 4 epitomizes the influence on the class of policy set by course coordinators. Students appeared to expect Professor Kale to negotiate on behalf of both the department and himself in regard to the amount of responsibility students would take for their actions (SAR-B) and effort (SAE). For example, after handing out a flyer (made by the course coordinators) announcing a Calculator Workshop, Professor Kale told students, "It is *strongly suggested* that you attend one of those days" (emphasis added). In response, several students raised different strands of negotiation. Students asked which calculators they could use, if they still had to attend the workshop if they used a calculator other than the one pictured on the flyer, and one student sought to avoid attending the workshop altogether. Professor Kale firmly maintained that attendance was "strongly suggested":

> *Kale*: (5:30 in Fig. 4) The purpose of this workshop is to make sure you have the necessary calculator skills. (TAR-SM 2)
> *Susan*: If we have an 85 [model number for a calculator], we can use it? (SAR-B 3)
> *Kale*: You can use the 85. The 85 and 86 are actually okay for this class. I think you can't have something like a TI-89. (TAR-B 3)
> *Milo*: Even if we have an 85 or 86, do we still go to this thing? (SAR-B 4)

One student asked, "what if we're not able to attend" (SAR-B 4) and, after Professor Kale answered, the student asked again, "what if, if we just absolutely can't?" (SAR-B 5). In each case, Kale responded by saying the workshop was "strongly suggested" and that he hoped his students would "try to find a way to attend." These Pattern B conversations also serve to illustrate Professor Kale's curricular values. He said in an interview that he believed students should be treated as "responsible adults who can evaluate a situation and make decisions about whether to follow policy, or not."

4.7 Pattern D: Escalation and resolution of conflict

Pattern D was an affectively charged variant of Pattern B that was distinct enough from it, especially by the end of the term, that we describe it separately. The dialogic nature for Patterns B and D are clearly negotiative, but Pattern D included more outside-of-class referents.

As an example, in the first 5 min shown in Fig. 6, eight different students spoke directly to Professor Kale or to each other and, for nearly a minute and a half, there were a dozen students speaking to each other at once about their concerns. In an interview, Professor Kale mentioned that it seemed to him that a conflict like this arose in his class weekly. This perception was supported by the fact that we found at least one Pattern D episode in seven of the ten class meetings we transcribed and coded. In Fig. 6, Pattern D began when a student asked about who wrote the exams. The interaction then moved into an environment where students spoke to each other, then moved back to the teacher directing discussion, and ended with Professor Kale responding to student comments by (as Kale put it): "taking responsibility for protecting the students from any potential harm caused by the course coordination." The class meeting in Fig. 6 was in week 14 of the 15-week class:

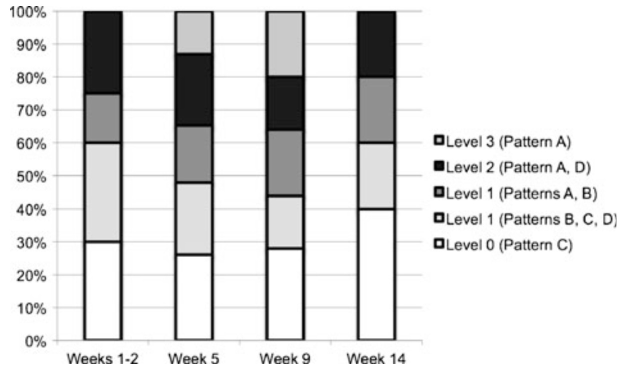
> *Jenna:* I was wondering who writes the [final] exam we take?
> *Kale:* Um, the exam actually already- They are not completed but I think they're in

the process of being written. The exams are written by, uh, a group of instructors for the course. I'm not one of the ones writing it. All sections have essentially just the same exam. You know, like at the testing center. So, all sections have a similar final-
George: That's so dumb.
Jenna: I was- I mean I'm just wondering. I'm not directing anything at you Just that, like, it seems like, especially after taking this exam [Exam 3] - and I do all the homework. I did the review. I went to the tutoring center before the last test to review anything I have questions on. Then I went to take the exam and when I looked at the exam, I went cross-eyed! [In the background a student, off-screen, laughs] because, like, some of the stuff I maybe remembered doing once, but it was not like our class specifically—
Israel: Of course.
Jenna:—no stress on that, but then, like, when we got tested on it, I just thought it was kind of unfair to us because we go over this stuff and I knew it. We learned this stuff and I almost bomb the test.
[Israel raises his hand mid-way through Jenna's last sentence, Kale nods at him, Israel speaks when Jenna finishes her sentence.]
Israel: Well, um, even taking the same classes with different teachers I found I learn more because my roommates study with me and we learn different things. Just because different teachers teach so many different methods and so many different formulas. I'll learn something I didn't even know. You use different or show different methods- So, when you compile, like, how many people take this class and how many different learning styles you have times how many different learning styles we all have it's completely, like, ludicrous that we all have to take the same test.
Kale: I'm not sure why the policy of the university is what it is. I have no comment. (pause) Uh, to be honest, I'd prefer to be able to write individual exams for individual classes.
Ruth: Yes, I mean we're not trying to say it's anything you've done, but you have your own way—

This pattern of heavily recursive *IRF* interaction centered on the common referent of the coordinator-generated exam policy. Professor Kale tabled the conversation shortly after Ruth's comment, with the promise to re-visit it at the next class meeting (which they did). Early in the semester, most of the Pattern B and Pattern D interactions between Professor Kale and students followed the one-step *IRF* pattern. As the semester progressed, the frequency of recursive *IRF* Pattern B and Pattern D negotiation increased, and that of one-stage *IRF* Pattern B and D interactions decreased. Also, as with the one detailed above, some of these negotiations stretched across class meetings.

Professor Kale said in interviews that he sought for extended conversation about mathematics content like those noted above around course coordination. He talked about this in post-class interviews, saying, "I just got tired, used up, dealing with the coordination stuff." As indicated in Fig. 8, the distribution of levels shifted over the semester. Professor Kale's process-oriented intended curricular values included supporting students' autonomy in mathematics learning. However, he saw his intention as conflicting with the departmental intention and, at times, with students' "coping" intention—expressed by students in interviews as a desire to "make it through the class" or "failure is not an option, I have to find a way to just pass."

Fig. 8 Frequency of discourse levels across the semester



4.8 The substitute: Mike the TA

The time-series of classroom interaction for a first year graduate student TA, Mike, substituting for Professor Kale is shown in Fig. 8. Based on additional observations and field notes in Mike's own classes, the pattern is also representative of interactions in the classes that Mike regularly taught.

In comparison to Professor Kale, the TA Mike was relying on a discourse Level 0 implementation of the traditional transmission model of teaching: "Here's some knowledge I have" (TAR-O is fully with the instructor); "Here's how I see it and you should see it" (TAR-SM fully to the instructor); "Do you see it?" At this point, the gray line showing discursive response goes from instructor utterances (above the horizontal line) to student utterances (below the line). Several students at once state the response they were told by Mike to offer:

Mike: You see (gesturing to graph projected by overhead transparency onto chalkboard). This side of the function is what? Going up. So it's increasing, and from this point to this point, the function is what? (pause)
Class: Decreasing.
Mike: Going down, so it's decreasing. From this point and up, it's what?
Class: Decreasing.
Mike: Decreasing. So if you take this point, (gestures to (−4, −3)), what is this point?
Class: [not in unison] Minus 4.
Mike: What is this point [again, gestures to graph where it passes through (−4, −3)]?
Class: Negative 3?
Mike: Negative 3. Clearly negative 4 is what? Less than negative 3.

In-class, Mike acknowledged the class' responses with a nod but did not probe for sense-making efforts on the part of the students. Towards the end of the class meeting, Mike asked students to work on three problems. The students all waited quietly until Mike gave the answers. Mike asserted in an interview that the most important thing for him was "to feel the students are following," adding, "they can figure out the details at home when they do their homework." Professor Kale commented on an early draft of this report that his own teaching was "probably like Mike's when [he] very first started out, more demonstration than conversation."

5 Discussion

Professor Kale approached instruction from the perspective of the process model, inviting students to negotiate social and sociomathematical norms and offering ways to take responsibility for sense-making. For Professor Kale, it was important that he have evidence of sense-making by his students from their discourse and problem-solving *during* class. Professor Kale's interactions with students, both the Level 0 and "virtual student" Level 1 type of communication of Pattern C and the Levels 1, 2, and 3 (in group work) interactions in Pattern A, gave evidence of an implemented curriculum aimed at mathematical meaning-making. The course design conflicted regularly with his efforts. In Patterns A and C, some of his intended curriculum was implemented. In Patterns B and D, however, Professor Kale felt compelled to state the departmental policies as "impersonally" as he could.

Professor Kale encouraged a classroom norm where he and his students built on an initial *IRF* interaction recursively, attempting to situate subsequent interaction in a referent context of mathematical concepts, cyclically revising the conversation around an original problem. In comparison, the TA Mike was steadfast in Level 0 interactions. Mike's interaction with students relied most on the first two parts, *IR*, of the *IRF* triad, with little follow-up to students' responses.

5.1 Patterns of discourse

In this report, our primary interest was to document the classroom discourse among students and instructor over the course of a semester of college algebra. The main question addressed here was:

> What is the nature of classroom discourse, and patterns in discourse, for this instructor in these two college algebra classes?

We explored Professor Kale's efforts to build and sustain sociomathematical and social norms and his concomitant grappling with course coordination environmental issues. In his work to support student autonomy and classroom discourse about mathematics, Professor Kale interspersed lecture chunks of Pattern C with episodes of the dialogic Pattern A. He and his classes operated at Levels 1 and 2 early in the term and then moved to include some student–student Level 3 interaction in small group work by week 9. However, this Level 3 discourse was sporadic, and by week 14, most of the Pattern A interactions were at Level 2 or 1. Professor Kale's interview comments suggest he did intend to have his students take more responsibility but had not yet determined how to support the physical and discourse space to do it given the norms asserted by the coordinators. Nonetheless, examination of Professor Kale's class meetings and interviews suggests he was engaging his students, in Pattern A, in "math-talk." His recognition of an earlier version of himself in the TA Mike, with entirely Level 0 interaction, suggests that Patterns C and A developed over many years for Professor Kale. Future research needs to explore whether and how this may be related to his process orientation and curricular values as well as on-the-job-learning of PCK.

5.2 Curricular values conflicts

Additionally, in reporting on the above, we addressed another research question:

> How does the professional environment, particularly course coordination, interact with classroom discourse?

In this report, we have identified conflicts evidenced in the classroom through student and instructor behaviors and in the evolution of the contract for them. Aspects of conflict fell into the three categories proposed by Bandura's (1986) social cognitive theory: personal, behavioral, and environmental. The environment established by the coordinators for college algebra was fraught with policies and changes in policy that led to conflict in Professor Kale's classes (Pattern D). At the behavior node was the conflict inherited into their classroom space from the larger college algebra coordination space: between the authoritarian role course coordinators expected lecturers to take and Professor Kale's preferred roles of presenter and facilitator. At the personal node for Professor Kale were the anger, frustration, stress, and hopelessness he experienced at different times during the semester as he attempted to construct a repertoire for working in the overlapping contexts that encompassed his classes (e.g., university, coordination, local culture of the students).

5.3 Pedagogical content knowledge

As with course coordination, though it was not the focus of this report, we addressed a third research question:

> How does the instructor's pedagogical content knowledge (PCK) reflect and get shaped by the classroom discourse in the classes he teaches?

The instructor's PCK, specifically that about his anticipations of what students knew and might struggle to understand or express, were evident in the categories of teacher's perceptions of student knowledge (TPK, Section 3.2.1) and attributions of effort by the teacher, (TAE, section 3.2.4). Additionally, Professor Kale's knowledge for action—about how to implement instruction given what he anticipated—relied on student sense-making (TAR-Sense-making, 3.2.3, Item 2). Notice that the teacher side of the see–saw of Pattern A (see Fig. 5) is largely made up of these three codes (TPK, TAR-Sense-making, and TAE). That is, his experiences and developed anticipations played a role in the types of discourse patterns offered to his class. Mike, a novice teacher, relied on a pattern that was almost exclusively TAR-Sense-making, without much incidence of the anticipatory-knowledge-related constructs of TPK or TAE. One potential area for further work is discerning what constellation of categories (and gradations of locus of control) might be appropriate targets for novice college instructor professional development.

5.4 Implications for further research

Research indicates that imagining the self one could become in new learning situations plays a key role in developing self-regulatory awareness for motivation and efficacy in resolving acculturative stress (Oyserman et al., 2002). Acculturation is the process of revising one's conceptions to allow for behaviors and personal views present in an environment where the contexting culture is different from one's own in significant ways. So, as one adept at mathematics learns about teaching college, especially in service courses, it may help to have multiple concrete, detailed models of the future instructor one might be. Resolving acculturative stress involves (a) conceiving of a possible future self who can operate fluidly in multiple cultures (Markus & Nurius, 1986) and (b) building intercultural competence for interactions across professional, social, and other cultural borders (for more on this emerging area of research in the US on K-12 teacher development, see DeJaeghere & Zhang, 2008, and references therein).

It is challenging to develop comfort and expertise in college teaching, particularly without any preparation in the pedagogy of adult learners. However, as Mason (2009) and others (Adams, 2002; Kung, 2010; Linse, Turns, Yellin, & VanDeGrift, 2004) have noted, a basic disconnect between the everyday world of university mathematics, guided by the imperative for logico-deductive theorems, and of the teaching world in college mathematics is that in teaching there are "too many factors connected with the setting, the individuals, the expectations, and the practices within lectures or tutorials to be able to declare one [practice] better than another universally" and that "seeking a mathematical-type of theorem with definitive conclusions" for what constitutes "best practice" is an exercise in futility (Mason, 2009, p. 5). Our goal here was to provide an accessible story that might serve as an imperfect mirror for researchers and practitioners of college mathematics. Existing professional development materials for collegiate mathematics offer teaching-activity-focused stories (e.g., about grading or setting policies, DeLong & Winter, 2002; Friedberg 2001). A challenge for future work is developing detailed and long-time scale versions of stories, like the story of Professor Kale, as reflective tools for self-awareness growth of college mathematics instructors.

References

Adams, K. A. (2002). *What colleges and universities want in new faculty.* Preparing Future Faculty Occasional Paper Number 10. Washington, DC: Association of American Colleges and Universities.

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning*. Stamford, CT: Ablex.

Bandura, A. (1986). *Social foundations of thought and action: A social cognitive theory.* New York: Prentice-Hall.

Bishop, A., FitzSimons, G., Seah, W. T., & Clarkson, P. (1999). *Values in mathematics education: Making values teaching explicit in the mathematics classroom*. Paper presented at the combined Annual Meeting of the Australian Association for Research in Education and the New Zealand Association for Research in Education, Melbourne, Australia, November 29-December 2, 1999. (ERIC Document Reproduction Service No. ED453075).

Cazden, C. B. (2001). *Classroom discourse: The language of teaching and learning*. Portsmouth, NH: Heinemann.

Davis, G. (1997). Mathematicians and the market. *Notices of the American Mathematical Society, 44*(10), 1307–1315.

Davis, M. K., Hauk, S., & Latiolais, M. P. (2009). Culturally responsive college level mathematics. In B. Greer, S. Nelson-Barber, A. P. & S. Mukhopadhyay (Eds.), *Culturally responsive mathematics education* (pp. 345–372). Mahwah, NJ: Erlbaum.

DeJaeghere, J. G., & Zhang, Y. (2008). Development of intercultural competence among US American teachers: Professional development factors that enhance competence. *Intercultural Education, 19*(3), 255–268.

DeLong, M., & Winter, D. (2002). *Learning to teach and teaching to learn mathematics*. Washington, DC: Mathematical Association of America.

Friedberg, S. (Ed.). (2001). *Teaching mathematics in colleges and universities: Case studies for today's classroom*. Providence, RI: American Mathematical Society.

Gee, J. P. (1996). *Social linguistics and literacies: Ideology in discourses*. London: Falmer Press.

Grundy, S. (1987). *Curriculum: Product or praxis?* Lewes, UK: Falmer.

Hauk, S. (2005). Mathematical autobiography among college learners in the United States. *Adults Learning Mathematics, 1*(1), 36–56.

Horn, L., Peter, K., Rooney, K., & Malizio, A. G. (2002). *National Center for Education Statistics' Profile of Undergraduates in U.S. Postsecondary Institutions: 1999–2000*. Published by the Office of Educational Research & Improvement, U.S. Dept. of Education. Publication No. NCES 2002–168.

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*, 81–116.

Kung, D. (2010). Teaching assistants learning how students think. In F. Hitt, D. Holton, & P. Thompson (Eds.), *Research in Collegiate Mathematics Education VII* (pp. 143–169). Providence, RI: AMS.

Lesh, R. (1996). Mathematizing: The “real” need for representational fluency. In L. Puig & G. Guitérrez (Eds.), *Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education, Addenda* (pp. 3–11). Valencia, Spain: Universitat de València.

Linse, A., Turns, J., Yellin, J. M. H., & VanDeGrift, T. (2004). Preparing future engineering faculty: Initial outcomes of an innovative teaching portfolio program. In *Proceedings of the 2004 ASEE Annual Conference and Exposition*. Salt Lake City, UT. (Session 3555. Retrieved January 30, 2005, from www.asee.org/acPapers/2004-1416_Final.pdf)

Markus, H., & Nurius, P. (1986). Possible selves. *The American Psychologist, 41*, 954–959.

Mason, J. (2009). Mathematics education: Theories, practice, and memories over fifty years. In B. Davis & S. Lerman (Eds.), *Mathematical action and structures of noticing: Studies on John Mason's contribution to mathematics education*. Rotterdam: Sense.

Nassaji, H., & Wells, G. (2000). What's the use of ‘triadic dialogue’? An investigation of teacher–student interaction. *Applied Linguistics, 21*(3), 376–406.

Ouellet, M. (Ed.). (2005). *Teaching inclusively: Diversity and faculty development*. Stillwater, OK: New Forums Press.

Oyserman, D., Terry, K., & Bybee, D. (2002). A possible selves intervention to enhance school involvement. *Journal of Adolescence, 25*, 313–326.

Ryve, A. (2011). Discourse research in mathematics education: A critical evaluation of 108 journal articles. *Journal for Research in Mathematics Education, 42*(2), 167–198.

Seymour, E., Melton, G., Wiese, D. J., & Pedersen-Gallegos, L. (2005). *Partners in innovation: Teaching assistants in college science courses*. Boulder, CO: Rowman & Littlefield.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–12.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*, 1022.

Truxaw, M. P., & Defranco, T. C. (2008). Mapping mathematics classroom discourse and its implications for models of teaching. *Journal for Research in Mathematics Education, 39*, 489–525.

Wertsch, J. V. (1998). *Mind as action*. New York: Oxford University Press.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*, 458–477.

Yackel, E., Rasmussen, C., & King, K. (2000). Social and sociomathematical norms in an advanced undergraduate mathematics course. *Journal of Mathematical Behavior, 19*(3), 275–287.